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1□□2021 □•□□□□□□□□□  $f(x) = ax^2 + 1(a > 0)$  □  $g(x) = x^2 + bx$  □

□1□□□□  $y = f(x)$  □□□  $y = g(x)$  □□□□□□ (1, 0) □□□□□□□□□  $a$  □  $b$  □□□

□2□□  $a^2 = 4b$  □□□□□□  $f(x) + g(x)$  □□□□□□□□□□□□□□□□  $(-\infty, -1]$  □□□□□□

□□□□□□□1□□ (1, 0) □□□□□□□  $f(x) = ax^2 + 1(a > 0)$  □

□  $f(x) = 2ax$  □  $k_1 = 2a$  □  $g(x) = x^2 + bx$  □

□  $g'(x) = 3x^2 + b$  □  $k_2 = 3 + b$  □  $\therefore 2a = 3 + b$  ①

□  $f'$  □1□  $= a + 1$  □  $g'$  □1□  $= 1 + b$  □  $\therefore a + 1 = 1 + b$  □□  $a = b$  □□□①□□□□  $\begin{cases} a = 3 \\ b = 3 \end{cases}$  □

□2□□  $a^2 = 4b$  □□□□□□  $h(x) = f(x) + g(x) = x^2 + ax^2 + \frac{1}{4}a^2x + 1$

□  $h(x) = 3x^2 + 2ax + \frac{1}{4}a^2$  □

□  $h(x) = 0$  □□□□  $x_1 = -\frac{a}{2}$  □  $x_2 = -\frac{a}{6}$  □

□  $a > 0$  □□□□  $-\frac{a}{2} < -\frac{a}{6}$  □

$\therefore$  □□□□□□  $(-\infty, -\frac{a}{2})$  □□□□□□□□□□□□□□□□  $(-\frac{a}{2}, -\frac{a}{6})$  □□□□□□□□□□□□□□□□  $(-\frac{a}{6}, +\infty)$  □□□□□□

① □  $-1, -\frac{a}{2}$  □□□□□□□□□□□□□□□□  $h(-1) = a - \frac{a^2}{4}$  □

② □  $-\frac{a}{2} < -1 < -\frac{a}{6}$  □□□□□□□□□□□□□□□□  $h(-\frac{a}{2}) = 1$

③ □  $-1 \leq -\frac{a}{6}$  □□□□□□□□□□□□□□□□  $h(-\frac{a}{2}) = 1$  □



$$\varphi(x) = -\frac{1}{2}x^2 - x^2 \ln x (x > 0), \varphi'(x) = -2x(1 + \ln x)$$

$$x \in (0, \frac{1}{e}) \quad \varphi'(x) > 0 \quad x \in (\frac{1}{e}, +\infty) \quad \varphi'(x) < 0$$

$$\varphi(x) \quad (0, \frac{1}{e}) \quad (\frac{1}{e}, +\infty)$$

$$\varphi(x)_{\max} = \varphi(\frac{1}{e}) = \frac{1}{2e^2}$$

$$f(x) = \frac{1}{2}x^2, g(x) = b \ln x, F(x) = f(x) - g(x)$$

$$F(x) \quad (0, 1] \quad b$$

$$b = e^{F(x)}$$

$$f(x) \quad g(x) \quad x \quad k \quad m \quad f(x) \dots kx + m \quad g(x) \dots kx + m \quad y = kx + m$$

$$f(x) \quad g(x) \quad b = e^{F(x)} \quad f(x) \quad g(x)$$

$$F(x) = f(x) - g(x) = \frac{1}{2}x^2 - b \ln x \quad F'(x) = x - \frac{b}{x} = \frac{x^2 - b}{x} (x > 0)$$

$$b, 0 \quad F(x) > 0 \quad F(x) \quad (0, 1]$$

$$F(x) \dots 0 \quad F(x) \quad (0, 1]$$

$$0 < b < 1 \quad F(x) \quad (0, \sqrt{b}) \quad (\sqrt{b}, 1) \quad x = \sqrt{b}$$

$$b \quad (0, 1)$$

$$b = e^{F(x)} \quad F(x) = \frac{1}{2}x^2 - e \ln x \quad F(x) = \frac{x^2 - e}{x}$$

$$0 < x < \sqrt{e} \quad F(x) < 0 \quad F(x)$$

$$x > \sqrt{e} \quad F(x) > 0 \quad F(x)$$

$$\square\square\square x=\sqrt{e}\square\square f(x)\square\square\square\square\square f(\sqrt{e})=0$$

$$\textcircled{2}\square\square\square\square\square\square f(x)\square g(x)\square\square\square\square x=\sqrt{e}\square\square\square\square\square\square(\sqrt{e},\frac{e}{2})\square$$

$$\square\square f(x)\square g(x)\square\square\text{“}\square\square\square\square\square\text{”}\square y-\frac{e}{2}=k(x-\sqrt{e})\square\square y=kx+\frac{e}{2}-k\sqrt{e}\square$$

$$\square\square f(x)\dots kx+\frac{e}{2}-k\sqrt{e}\square\square x\in R\square\square\square\square\square\square$$

$$\square\square x^2-2kx-\textcolor{red}{e}+2k\sqrt{e}\textcolor{red}{.0}\square\square\square\square\square\square\square$$

$$\square\square\triangle=4k^2-4(2k\sqrt{e}-\textcolor{red}{e})=4k^2-8k\sqrt{e}+4e=4(k-\sqrt{e})^2\textcolor{red}{,0}\square$$

$$\square\square k=\sqrt{e}$$

$$\square\square\square\square\square g(x)\textcolor{red}{,}\sqrt{e}x-\frac{e}{2}(x>0)\square\square\square\square\square$$

$$\square\square G(x)=\textcolor{red}{e}lnx-\sqrt{e}x+\frac{e}{2}\square\square G(x)=\frac{e}{x}-\sqrt{e}=\frac{\sqrt{e}(\sqrt{e}-x)}{x}\square$$

$$\square\square\square 0< x<\sqrt{e}\square\square G(x)>0\square\square x>\sqrt{e}\square\square G(x)<0$$

$$\square\square x=\sqrt{e}\square\square G(x)\square\square\square\square\square\square 0\square\square g(x)\textcolor{red}{,}\sqrt{e}x-\frac{e}{2}(x>0)\square\square\square\square\square$$

$$\square\square f(x)\square g(x)\square\square\text{“}\square\square\square\square\square\text{”}\square y=\sqrt{e}x-\frac{e}{2}\square$$

$$4\square\square 2021\square\bullet\square\square\square\square\square\square\square\square f(x)=a^2x^2(a>0)\square g(x)=\sqrt{9-(x-b)^2}\square$$

$$\square 1\square\square\square\square y=f(x)\square\square\square\square\square\square\square\square x-y-3=0\square\square\square\square\square\square\square\square\sqrt{2}\square\square a\square\square\square\square$$

$$\square 2\square\square\square x\square\square\square\square (x-1)^2>f(x)\square\square\square\square\square\square\square\square\square 3\square\square\square\square\square a\square\square\square\square\square\square\square$$

$$\square 3\square\square\square\square\square f(x)\square g(x)\square\square\square\square\square\square\square\square\square x\square\square\square\square\square\square k\square m\square\square\square f(x)\dots kx+m\square g(x)\textcolor{red}{,}kx+m\square\square\square\square\square\square\square\square y=kx+m\square\square$$



$$x = \sqrt{3} \quad \left( \sqrt{3}, \frac{3}{2} \right)$$

$$y = f(x) \quad y - \frac{3}{2} = k(x - \sqrt{3})$$

$$f(x) \quad f(x) = x$$

$$k = \sqrt{3}$$

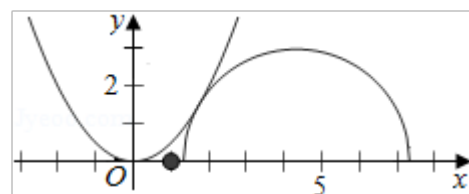
$$\sqrt{3}x - y - \frac{3}{2} = 0$$

$$g(x) \quad \left( \frac{5}{2}\sqrt{3}, 0 \right) \quad 3$$

$$d = \frac{|\sqrt{3} \cdot \frac{5\sqrt{3}}{2} - \frac{3}{2}|}{\sqrt{3+1}} = 3$$

$$y = g(x)$$

$$y = \sqrt{3}x - \frac{3}{2} \quad f(x) \quad g(x)$$



$$f(x) = a^2 x^2 \quad (a > 0) \quad g(x) = \ln x$$

$$y = f(x) \quad x - y - 3 = 0 \quad 2\sqrt{2} \quad a$$

$$f(x) \quad g(x) \quad x \quad k \quad m \quad f(x) \dots kx + m \quad g(x) \dots kx + m \quad y = kx + m$$

$$f(x) \quad g(x) \quad a = \frac{\sqrt{2}}{2} \quad b = e \quad f(x) \quad g(x)$$

$$f(x) = a^2 x^2 \quad f(x) = 2a^2 x$$

$$\square f(x)=2a^2x=1$$

$$\square\square x=\frac{1}{2a^2}\square$$

$$\square\square y=\frac{1}{4a^2}\square$$

$$\square\square(\frac{1}{2a^2}-\frac{1}{4a^2})\square\square\square x-y-3=0\square\square\square\square 2\sqrt{2}\square$$

$$\square\frac{|\frac{1}{2a^2}-\frac{1}{4a^2}-3|}{\sqrt{2}}=2\sqrt{2}\square$$

$$\square\square\square a=\frac{\sqrt{7}}{14}\square$$

$$\square2\square\square F(x)=f(x)-g(x)=\frac{1}{2}x^2-\ln x\square$$

$$\square F(x)=x-\frac{e}{x}\square$$

$$\square\square\square 0< x<\sqrt{e}\square\square F(x)<0\square\square x>\sqrt{e}\square\square F(x)>0\square$$

$$\square\square x=\sqrt{e}\square\square F(x)\square\square\square\square\square 0\square$$

$$\square f(x)\square g(x)\square\square\square\square x=\sqrt{e}\square\square\square\square\square(\sqrt{e}-\frac{1}{2}e)\square$$

$$\square f(x)\square g(x)\square\square“\square\square\square”\square$$

$$\square\square\square y-\frac{1}{2}e=k(x-\sqrt{e})\square\square y=kx+\frac{1}{2}e-k\sqrt{e}\square$$

$$f(x) = kx + \frac{1}{2}e^{-k\sqrt{e}}$$

$$x^2 - 2kx - e + 2k\sqrt{e} = 0$$

$$\Delta = 4k^2 - 4(2k\sqrt{e} - e) = 4k^2 - 8k\sqrt{e} + 4e = 4(k - \sqrt{e})^2 \geq 0$$

$$k = \sqrt{e}$$

$$g(x) = \sqrt{e}x - \frac{1}{2}e \quad (x > 0)$$

$$G(x) = e \ln x - x\sqrt{e} + \frac{1}{2}e \quad (x > 0)$$

$$G(x) = \frac{e}{x} - \sqrt{e} = \frac{\sqrt{e}(\sqrt{e} - x)}{x}$$

$$0 < x < \sqrt{e} \implies G(x) > 0 \quad x > \sqrt{e} \implies G(x) < 0$$

$$x = \sqrt{e} \implies G(x) = 0 \quad g(x) = \sqrt{e}x - \frac{1}{2}e$$

$$y = \sqrt{e}x - \frac{1}{2}e$$

$$f(x) = \frac{2x^2}{e} + \frac{e^2}{x} \quad g(x) = 3e \ln x$$

$$f(x)$$

$$f(x) = ax + b \quad g(x) \quad x \in (0, +\infty)$$

$$f(x) = \frac{2x^2}{e} + \frac{e^2}{x}$$



$$f(x) = \frac{4x}{e} - \frac{e^3}{x^2} = \frac{4x^3 - e^3}{ex^2}$$

$$f(x) = 0 \quad x = \frac{e}{\sqrt[3]{4}}$$

$$x < \frac{e}{\sqrt[3]{4}} \quad x \neq 0 \quad f(x) < 0 \quad x > \frac{e}{\sqrt[3]{4}} \quad f(x) > 0$$

$$f(x) \quad (-\infty, 0) \quad (0, \frac{e}{\sqrt[3]{4}}) \quad (\frac{e}{\sqrt[3]{4}}, +\infty)$$

$$f'(x) = g(x) = 3e - \frac{ae}{x^2} \quad b = 3e - \frac{ae}{x^2}$$

$$ax + b \cdot g(x) = a(x - e) - 3e(1 - \ln x) \cdot 0$$

$$h(x) = a(x - e) + 3e(\ln x - 1) \quad h(x) = a - \frac{3e}{x} \quad (x > 0)$$

$$a, 0 \quad h(x) < 0 \quad h(x) \quad (0, +\infty) \quad x > e \quad h(x) < h(e) = 0$$

$$a > 0 \quad h(x) \quad (0, \frac{3e}{a}) \quad (\frac{3e}{a}, +\infty)$$

$$h(x) \quad (0, +\infty) \quad h(\frac{3e}{a}) = 3e(2 - \ln \frac{3e}{a}) - ae \cdot 0$$

$$m(a) = 3e(2 - \ln \frac{3e}{a}) - ae \quad m(a) = \frac{3e}{a} - e \quad m(a) = 0 \quad m(a) = 0$$

$$m(a) = 0 \quad a = 3 \quad b = 0$$

$$a = 3 \quad b = 0 \quad f(x) = ax + b \cdot \frac{2x^2}{3} + \frac{e^3}{x} \cdot 3x \Leftrightarrow \frac{2x^3 - 3ex^2 + e^3}{ex} \quad (x > 0) \Leftrightarrow 2x^3 - 3ex^2 + e^3 = 0$$

$$\varphi(x) = 2x^3 - 3ex^2 + e^3 \quad \varphi'(x) = 6x(x - e) \quad \varphi(x) \quad (0, +\infty) \quad \varphi(e) = 0 \quad \varphi(x) = 0$$

$$a = 3 \quad b = 0 \quad f(x) = ax + b \cdot g(x) \quad x \in (0, +\infty)$$

7月2021•  
 $f(x) = \frac{x^2}{2e} - ax, g(x) = \ln x - ax, a \in R$

1.  $f(x) \geq 0$

2.  $f(x) \geq g(x)$

3.  $a \leq b$   $f(x) \geq g(x)$   $x > 0$   $a \leq b$

1.  $a = 0$   $f(x) = \frac{x^2}{2e}$   $f(x) \geq 0$   $\{0\}$

$a \neq 0$   $f(x) = \frac{x^2}{2e} - a$

$a > 0$   $f(x) \geq 0$   $[0, 2ae]$

$a < 0$   $f(x) \geq 0$   $[2ae, 0]$

$a = 0$   $f(x) \geq 0$   $\{0\}$

$a > 0$   $f(x) \geq 0$   $[0, 2ae]$

$a < 0$   $f(x) \geq 0$   $[2ae, 0]$  ... 4

2.  $h(x) = f(x) - g(x) = \frac{x^2}{2e} - \ln x$   $h(x) = \frac{x}{e} - \frac{1}{x} = \frac{x^2 - e}{ex}$

$h(x) = 0$   $x = \sqrt{e}$

$x$	$(0, \sqrt{e})$	$\sqrt{e}$	$(\sqrt{e}, +\infty)$
$h(x)$	-	0	+
$h(x)$	$\searrow$		$\nearrow$

$$h(x) = h(\sqrt{e}) = 0$$

$$h(x) = \frac{x^2}{2e} - \ln x, 0 < x < \sqrt{e}$$

$$h(x) = \frac{x^2}{2e} - \ln x, x > \sqrt{e}$$

$$h(x) = \frac{x^2}{2e} - \ln x, x > 0$$

$$h(x) = \frac{x^2}{2e} - \ln x, x > 0$$

$$h(x) = \frac{x^2}{2e} - \ln x, x > 0$$

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$$h(x) = \frac{x^2}{2e} - \ln x, x > 0$$

$$h(x) = \frac{x^2}{2e} - \ln x, x > 0$$

$$\square\square\square\square \quad a=\frac{1}{2\sqrt{e}} \quad b=-\frac{1}{2} \quad \square\square\square\square\square\square \quad \square\square16 \quad \square\square$$

$$8\square\square2021\bullet\square\square\square\square\square\square\square\square\square\square\square\square \quad f(x)=e^x-\frac{x+1}{x-1} \quad \square$$

$$\square1\square\square\square \quad f(x) \quad \square\square\square\square\square\square\square\square \quad f(x) \quad \square\square\square\square \quad 2 \quad \square\square\square\square$$

$$\square2\square\square \quad x_0 \quad f(x) \quad \square\square\square\square\square\square\square\square\square\square \quad y=e^x \quad \square\square \quad A(x_0 \quad e^{x_0}) \quad \square\square\square\square\square\square\square\square \quad y=\ln x \quad \square\square\square\square$$

$$\square\square\square\square\square\square\square1\square \quad f(x) \quad \square\square\square\square\square \quad (-\infty,1) \cup (1,+\infty) \quad \square$$

$$\square \quad f(x)=e^x+\frac{2}{(x-1)^2}>0 \quad \square$$

$$\therefore \square\square \quad f(x) \quad \square \quad (-\infty,1) \quad \square \quad (1,+\infty) \quad \square\square\square\square\square\square$$

$$\square \quad f(-1)=\frac{1}{e}>0, \quad (-2)=\frac{1}{e}-\frac{1}{3}<0 \quad \square$$

$$\therefore f(x) \quad \square \quad (-\infty,1) \quad \square\square\square\square\square\square \quad x \in (-2,-1) \quad \square\square \quad f(x)=0, e^x=\frac{x_1+1}{x_1-1} \quad \square$$

$$\square \quad 1<-x_1<2, f(-x_1)=e^x-\frac{-x_1+1}{-x_1-1}=\frac{x_1-1}{x_1+1}+\frac{-x_1+1}{x_1+1}=0 \quad \square$$

$$\square \quad f(x) \quad \square \quad (1,+\infty) \quad \square\square\square\square\square\square \quad -x_1 \quad \square$$

$$\square\square\square \quad f(x) \quad \square\square\square\square\square\square\square\square\square$$

$$\square2\square\square\square\square\square\square \quad -x_0=\ln e^{x_0} \quad \square\square\square \quad B(e^{x_0},-x_0) \quad \square\square\square \quad y=\ln x \quad \square\square$$

$$\square\square\square\square \quad f(x_0)=0 \quad \square\square \quad e^{x_0}=\frac{x_0+1}{x_0-1} \quad \square$$

$$\square\square\square \quad AB \quad \square\square\square\square \quad \square \quad k=\frac{-x_0-e^{x_0}}{e^{x_0}-x_0}=\frac{-x_0-\frac{x_0+1}{x_0-1}}{\frac{x_0-1}{x_0+1}-x_0}=\frac{x_0+1}{x_0-1}=e^{x_0}$$





$$\lim_{x \rightarrow +\infty} f(x) = 0$$

$$f(x) = 0 \quad x > 1$$

$$0 < \frac{1}{x} < 1 \quad f\left(\frac{1}{x}\right) = \ln\left(\frac{1}{x}\right) = -\ln x = -f(x)$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1}{x} = +\infty$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} x \cdot x_2 = 1$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x+1}{x-1}$$

$$y = \ln x \quad x = x_0 \quad y = \frac{1}{x_0}$$

$$y = \ln x \quad (x_0, \ln x_0) \quad l_1: y = \frac{1}{x_0}(x - x_0) + \ln x_0 \quad \ln x_0 = \frac{x_0 + 1}{x_0 - 1} \quad l_1: y = \frac{x}{x_0} + \frac{2}{x_0 - 1}$$

$$y = e^x \quad \frac{1}{x_0}$$

$$e^x = \frac{1}{x_0} \quad x = -\ln x_0$$

$$\left(-\ln x_0, \frac{1}{x_0}\right)$$

$$\left(-\ln x_0, \frac{1}{x_0}\right) \quad y = e^x \quad l_2: y = \frac{1}{x_0}(x + \ln x_0) + \frac{1}{x_0}$$

$$\ln x_0 = \frac{x_0 + 1}{x_0 - 1} \quad l_2: y = \frac{x}{x_0} + \frac{2}{x_0 - 1}$$

$$I_1 \cap I_2 \neq \emptyset$$

$$y = \frac{1}{x_0} x + \ln x_0 - 1 \quad y = \ln x \quad y = e^x$$

$$11 \text{ 年 } 2021 \bullet \text{ 设 } f(x) \text{ 与 } g(x) \text{ 在 } x_0 \in R \text{ 处满足 } f(x_0) = g(x_0) \text{ 且 } f'(x_0) = g'(x_0)$$

$$x_0 \text{ 处 } f(x) \text{ 与 } g(x) \text{ 有公切线 } S$$

$$1 \text{ 设 } f(x) = x, g(x) = x^2 + 2x - 2 \text{ 有公切线 } S$$

$$2 \text{ 设 } f(x) = ax^2 - 1, g(x) = \ln x \text{ 有公切线 } S \text{ 求 } a$$

$$3 \text{ 设 } f(x) = -x^2 + a, g(x) = \frac{be^x}{x} \text{ 在 } (0, +\infty) \text{ 有公切线 } S$$

解法

$$1 \text{ 设 } f(x) = 1, g(x) = 2x + 2$$

$$\begin{cases} x = x^2 + 2x - 2 \\ 1 = 2x + 2 \end{cases} \Rightarrow f(x) = x, g(x) = x^2 + 2x - 2 \text{ 有公切线 } S$$

$$2 \text{ 设 } f(x) = 2ax, g(x) = \frac{1}{x}, x > 0$$

$$f(x) = g(x) \Rightarrow \frac{1}{x} = 2ax \Rightarrow x = \sqrt{\frac{1}{2a}}$$

$$f'(\sqrt{\frac{1}{2a}}) = -\frac{1}{2} = g'(\sqrt{\frac{1}{2a}}) = -\frac{1}{2} \ln 2 \Rightarrow a = \frac{e}{2}$$

$$3 \text{ 设 } f(x) = -2x, g(x) = \frac{be^{x-1}}{x^2} (x \neq 0)$$

$$f(x_0) = g(x_0) \Rightarrow b > 0 \Rightarrow be^{x_0} = -\frac{2x_0^2}{x_0 - 1} > 0 \Rightarrow 0 < x_0 < 1$$



$$\square \quad f(x_0)=g(x_0) \quad \square \square \quad -x_0^2+a=\frac{be^{x_0}}{x_0}=-\frac{2x_0^2}{x_0-1} \quad \square \square \quad a=x_0^2-\frac{2x_0^2}{x_0-1} \quad \square$$

$$\square \quad h(x)=x^2-\frac{2x^2}{x-1}-a=\frac{-x^2+3x^2+ax-a}{1-x} \quad \square \quad (a>0,0<x<1) \quad \square$$

$$\square \quad m(x)=-x^2+3x^2+ax-a \quad \square \quad (a>0,0<x<1) \quad \square$$

$$\square \quad m(0)=-a<0 \quad \square \quad m(1)=2>0 \quad \square \square \quad m(0)m(1)<0 \quad \square$$

$$\square \quad m(x) \quad \square \square \square \square \quad (0,1) \quad \square \square \square \square \square$$

$$\square \quad m(x) \quad \square \quad (0,1) \quad \square \square \square \square \square$$

$$\square \quad h(x) \quad \square \quad (0,1) \quad \square \square \square \square \square$$

$$\square \square \square \quad b>0 \quad \square \square \quad f(x) \quad \square \quad g(x) \quad \square \square \square \quad (0,+\infty) \quad \square \square \square \quad “S” \quad \square \square$$

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